

Such a comparison I have endeavoured to make, or rather to indicate the basis on which it may be made, so far as systems of geometrical algebra are concerned. As a contribution to analysis in general, I suppose that there is no question that Grassmann's system is of indefinitely greater extension, having no limitation to any particular number of dimensions.

J. WILLARD GIBBS.

The Flying to Pieces of a Whirling Ring.

IN NATURE of May 14 (p. 31) I notice a letter by Mr. C. A. Carus-Wilson on the rotation of a hollow steel flask, composed apparently of a spherical shell mounted on an axis constituting a diameter. Mr. Carus-Wilson speaks of this body as being under a "tension" of "31.5 tons per square inch" at a certain speed of rotation. He does not, however, specify what is the tension to which he refers, nor where it is found, neither does he give the density and elastic constants of the material nor indicate the method by which he arrived at his result.

So far as I know, the only problem of the kind which has yet been solved is that of an isotropic spherical shell¹ rotating about an imaginary axis through its centre at speeds at which the strains follow Hooke's law. This differs from the case Mr. Carus-Wilson speaks of, inasmuch as the existence of a real material axis must introduce conditions somewhat different from those assumed by the mathematical theory, and further the results obtained by this theory cannot legitimately be applied to speeds exceeding that where bulging becomes sensible, if indeed so far.

This solution is probably, however, the nearest to the practical problem at present attainable.

According to it the strains and stresses vary throughout the shell with the distance from the centre, and the angular distance from the axis of rotation. They also depend on the density and on the elastic properties of the material. There are also at every point three principal stresses, whereof one it is true vanishes over the surfaces. Thus such a statement as Mr. Carus-Wilson's requires further explanation.

According to the two theories most commonly entertained, the quantity which determines the limiting safe speed is the maximum value of either the *greatest strain* or the *maximum stress-difference*,—i.e. the algebraical difference between the greatest and least principal stresses at a point. Over the surfaces of the shell the absolutely greatest values of both these quantities are found, for shells of all degrees of thickness, in the equatorial plane—or plane through the centre perpendicular to the axis of rotation.

Denoting the angular velocity by ω , the radii of the outer and inner surfaces respectively by a and a' , the density by ρ , Young's modulus by E , the greatest strain by s , the maximum stress-difference by S , and the stress at right angles to the meridian plane by Φ , the three last quantities being measured in the equator, the following are some of the results I found for materials in which Poisson's ratio is $1/4$:—

	$Es/\omega^2\rho a^2$		$S/\omega^2\rho a^2$		$\Phi/\omega^2\rho a^2$	
	Inner surface.	Outer surface.	Inner surface.	Outer surface.	Inner surface.	Outer surface.
$a'/a = 0.9$	0.950	0.833	1.064	0.866	0.912	0.866
$\frac{a-a'}{a}$ negligible	1.0	1.0	1.0	1.0	1.0	1.0

Apparently in the case mentioned by Mr. Carus-Wilson, $a'/a = 15/16 = 0.9375$. Supposing the material to have Poisson's ratio = $1/4$, which seems to accord fairly with experiments on steel, the approximate values of s , S , and Φ , for this case could be obtained by interpolation from those I give above. The differences between the values of corresponding strains and stresses at the two surfaces are less, of course, for $a'/a = 15/16$ than for $a'/a = 0.9$, but still are far from negligible. Mr. Carus-Wilson's numerical result rather suggests that the tension he refers to is the stress Φ , measured as above in the equator, and that he employed the formula $\Phi = \omega^2\rho a^2$. This formula (see Cambridge Philosophical Transactions, vol. xiv. p. 300), is correct for the value of Φ in the equator in an infinitely thin shell, but it does not strictly apply to any shell whose thickness is comparable with its radius. In the paper in the Cambridge Transactions first referred to, there are given tables of the numerical measures of the strains and stresses over the surfaces for a series of values

of a'/a for materials in which Poisson's ratio is $1/4$. These give by interpolation fairly accurate values for all values of a'/a . For other values of Poisson's ratio, recourse must be had to the general formulæ given in the paper, unless $\epsilon \equiv 1 - a'/a$, is very small, when the greatest values of s and S are given approximately by $Es/\omega^2\rho a^2 = 1 - \frac{1}{2}\epsilon(1 - \eta)$, $S/\omega^2\rho a^2 = 1 + \epsilon/(1 + \eta)$, where η is Poisson's ratio (see Camb. Trans., vol. xiv. p. 304). May 16.

C. CHREE.

A Comet observed from Sunrise to Noon.

A SHORT time ago I got the loan of an old number of *Harper's Monthly* (March 1889), good reading matter being very acceptable, however old, in this outlandish place, in which I read an article, on the origin of celestial species, by J. Norman Lockyer, F.R.S., Cor. Inst. France, that set me thinking of what I observed of the great comet of 1882, when it made its tremendous plunge round the sun, on September 18. At that time I was master of a small vessel, trading in the Society Islands; and on the day mentioned—in latitude $16^\circ 25'$ S., longitude $151^\circ 57'$ W. of Greenwich, a position about midway between the two islands Bolabola and Maupiti (the Maurua of Cook)—I saw, with the naked eye, the comet travel about 90° of the circle of the sun's disk, between sunrise and noon; but what made it most remarkable to us was that it should be possible for us, in a perfectly clear sky, to be able to watch it all, from sunrise to noon, with very little more distress to the eye than if in a clear night looking at a full moon.

Now, Sir, may it not be that this is partly a *proof* of the theory set forth by Norman Lockyer in the article above mentioned, viz. that comets are swarms of meteorites in collision, travelling through space, and that the outer invisible part of the swarm that formed this comet's nucleus had partially eclipsed the sun, like a veil over it? I am not aware if it was noticed by any competent astronomer or not, but the chances are that none had the splendid opportunity that we had to see the phenomena; so, Sir, knowing that men of science are always glad to get facts from observers in all parts of the world is my excuse for writing this to you, not knowing Mr. Lockyer's address. Thinking this, although late, may probably be of some interest to the scientific world, I leave you to do what you may think proper with it.

WM. ELLACOTT.

Raiatea, January 30.

Graphic Daily Record of the Magnetic Declination or Variation of the Compass at Washington.

I BEG to call your attention to the enclosed reprint from the May Pilot Chart of curves of magnetic declination as recorded at the United States Naval Observatory at Washington. This reprint admits of reproduction more readily than the curves as shown on the Pilot Chart, being in black and white, and only reduced to two-fifths of true size (the reduction on the Pilot Chart itself being one-quarter). It will be interesting to this Office to elicit expressions of opinion relative to the advantages of the prompt publication of these curves. The experiment is to be tried for three months, but it is not likely to be continued longer unless certain decided advantages develop. It may be of sufficient interest to NATURE to republish these curves, and thus assist us in giving them wide publicity.

RICHARDSON CLOOER,
Hydrographer.

Washington, D.C., May 6.

[We are unable to print the curves, but we may note that they are issued with the following explanation :—“ These curves indicate graphically the true direction in which the magnetic needle at the Naval Observatory pointed during each instant from noon, March 29, to noon, April 30. The base-line shows a slight break at the end of each two hours, 75th meridian time, and the amount of westerly variation at any time is 4° plus the number of minutes represented by the height of the curve above the base line at that time, measured by the scale at the right or left margin of the diagram. The slight breaks in the curve itself occur when the chronograph sheets are changed. Although the daily change of variation at any one place, even in magnetic storms such as those that have occurred during the past month, is too small to be of any importance in practical navigation, yet it is thought that the prompt publication of these curves cannot fail to interest masters of vessels, as well as scientific men. The mean daily curve, which can be drawn by taking the average of many such curves, shows that there is a regular, though slight,

¹ Cambridge Philosophical Society's Transactions, vol. xiv. pp. 467-483.

daily change in the variation, somewhat analogous to the daily range of the barometer, although the daily minimum of variation at Washington occurs at about 8 a.m., and the maximum between 1 and 2 p.m. It is proposed to continue the publication of these curves on this Chart for at least three months, and any questions regarding them will receive immediate consideration and reply. The attention of masters of vessels is called to the form issued by this Office for the record of observations of variation at sea, and to the general importance of the subject in connection with vessels' compasses and the variation curves plotted on our charts."]

The Alpine Flora.

IN connection with this subject (see NATURE, vol. xliii. p. 581) it may be well to draw the attention of botanists to the fact that a young vigorous strawberry plant, in an exposed garden, will, during the winter season, place all its leaves in a perfectly horizontal position, some even close to and resting on the ground, in striking contrast to its summer habit of erect growth, whereby it is often damaged by strong winds.

Whether direct climatal conditions be the sole cause of this peculiarity, or whether inherited, I cannot determine; presumably, in its natural surroundings, the continual crowding and consequent struggle would not necessitate the adoption of dwarfing as a means of survival.

J. LOVEL.

May 13.

Magnetic Anomalies in Russia.

THE magnetic disturbances in England and Wales as communicated to NATURE, vol. xliii. p. 617, by M. Mascart and A. W. Rücker, are of great interest, but the size of the disturbances between Charkov and Kursk in Russia is of much higher value. More than 150 stations with magnetic elements have proved that in the above region there are points where the declination differs by 86°, the inclination by 29°, and the magnetic total force by 0.39 el. un. The principal centres are distant from each other not more than 12 kilometres. The m. elements are:—

Principal centres of disturbance.	Decl.	Incl.	Total force, c. u.
Nepchaevo	+ 48°	+ 81°	0.84
Visloe	- 33°	+ 52°	0.65
Kisselevo	- 38°	+ 63°	0.72
Sobinino	+ 30°	+ 60°	0.75
Petrovavlovka	- 20°	+ 76°	0.80
Belgorod	- 36°	+ 71°	0.64

The normal values are - 1° Decl.; + 64° Incl.; 0.48 total force. The districts are covered by sedimentary rocks.

St. Petersburg, April 30.

A. DE TILLO.

THE REJUVENESCENCE OF CRYSTALS.¹

VERY soon after the invention of the microscope, the value of that instrument in investigating the phenomena of crystallization began to be recognized.

The study of crystal-morphology and crystallogeneses was initiated in this country by the observations of Robert Boyle; and since his day a host of investigators—among whom may be especially mentioned Leeuwenhoek and Vogelsang in Holland, Link and Frankenheim in Germany, and Pasteur and Senarmont in France—have added largely to our knowledge of the origin and development of crystalline structures. Nor can it be said with justice that this field of investigation, opened up by English pioneers, has been ignobly abandoned to others; for the credit of British science has been fully maintained by the numerous and brilliant discoveries in this department of knowledge by Brewster and Sorby.

There is no branch of science which is more dependent for its progress on a knowledge of the phenomena of crystallization than geology. In seeking to explain the complicated phenomena exhibited by the crystalline masses composing the earth's crust, the geologist is

constantly compelled to appeal to the physicist and chemist; from them alone can he hope to obtain the light of experiment and the leading of analogy, whereby he may hope to solve the problems which confront him.

But if geology owes much to the researches of those physicists and chemists who have devoted their studies to the phenomena of crystallization, the debt has been more than repaid through the new light which has been thrown on these questions by the investigation of naturally-formed crystals by mineralogists and geologists.

In no class of physical operations is *time* such an important factor as in crystallization; and Nature, in producing her inimitable examples of crystalline bodies, has been unsparing in her expenditure of time. Hence it is not surprising to find that some of the most wonderful phenomena of crystallization can best be studied—some, indeed, can only be studied—in those exquisite specimens of Nature's handiwork which have been slowly elaborated by her during periods which must be measured in millions of years.

I propose to-night to direct your attention to a very curious case in which a strikingly complicated group of phenomena is presented in a crystalline mass; and these phenomena, which have been revealed to the student of natural crystals, are of such a kind that we can scarcely hope to reproduce them in our test-tubes and crucibles.

But if we cannot expect to imitate all the effects which have in this case been slowly wrought out in Nature's laboratory, we can, at least, investigate and analyze them; and, in this way, it may be possible to show that phenomena like those in question must result from the possession by crystals of certain definite properties. Each of these properties, we shall see, may be severally illustrated and experimentally investigated, not only in natural products, but in the artificially-formed crystals of our laboratories.

In order to lead up to the explanation of the curious phenomena exhibited by the rock-mass in question, the first property of crystals to which I have to refer may be enunciated as follows:—

Crystals possess the power of resuming their growth after interruption; and there appears to be no limit to the time after which this resumption of growth may take place.

It is a familiar observation that if a crystal be taken from a solution and put aside, it will, if restored after a longer or shorter interval to the same or a similar solution, continue to increase as before. But geology affords innumerable instances in which this renewal of growth in crystals has taken place after millions of years must have elapsed. Still more curious is the fact, of which abundant proof can be given, that a crystal formed by one method may, after a prolonged interval, continue its growth under totally different conditions and by a very different method. Thus, crystals of quartz, which have clearly been formed in a molten magma, and certain inclosures of glass, may continue their growth when brought in contact with solutions of silica at ordinary temperatures. In the same way, crystals of felspar, which have been formed in a mass of incandescent lava, may increase in size, when solvent agents bring to them the necessary materials from an enveloping mass of glass, even after the whole mass has become cold and solid.

It is this power of resuming growth after interruption, which leads to the formation of zoned crystals, like the fine specimen of amethyst enclosed in colourless quartz, which was presented to the Royal Institution seventy years ago by Mr. Snodgrass.

The growth of crystals, like that of plants and animals, is determined by their environment; the chief conditions affecting their development being temperature, rate of growth, the supply of materials (which may vary in

¹ The Friday Evening Discourse, delivered at the Royal Institution on January 30, 1891, by Prof. John W. Judd, F.R.S.